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DEGRADATION IN NOISE FIGURE ALONG A CHAIN OF NOISY NETWORKS.(U)
JAN 76 H J SHAW N00014-75-C-0632
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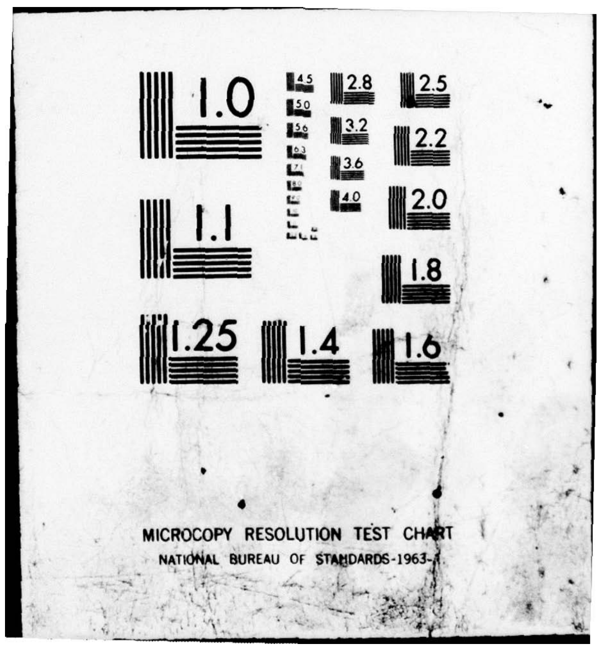
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(6) DEGRADATION IN NOISE FIGURE ALONG A CHAIN OF NOISY NETWORKS.

by

(10) H. J. Shaw

(1)



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Microwave Laboratory

W. W. Hansen Laboratories of Physics

Stanford University

Stanford, California

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Consider the case of a network as in Fig. 1, having input consisting of a coherent signal power $S_{in}$ plus a noise power $N_{in}$ .

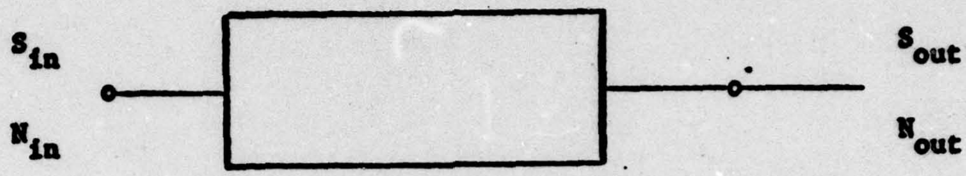


FIG. 1--Notation for noisy network.

The network is defined by a noise figure ratio $F$ and a power gain ratio $G$ . $G$ may be $> 1$ (net gain) or $< 1$ (net attenuation).

The input noise $N_{in}$ consists of two components

$$N_{in} = T + \Delta N_{in} \quad . \tag{1}$$

Here $T = kT\cdot\Delta f$ is the Johnson noise level and $\Delta N$ is the "excess" noise above Johnson noise.

After going through the network of Fig. 1 the output noise is

$$N_{out} = FGT + G\,\Delta N_{in} \quad . \tag{2}$$

That is, the Johnson noise component is multiplied by $FG$ , while the excess noise is just multiplied by $G$ .

Consider a tandem array of networks as in Fig. 2.

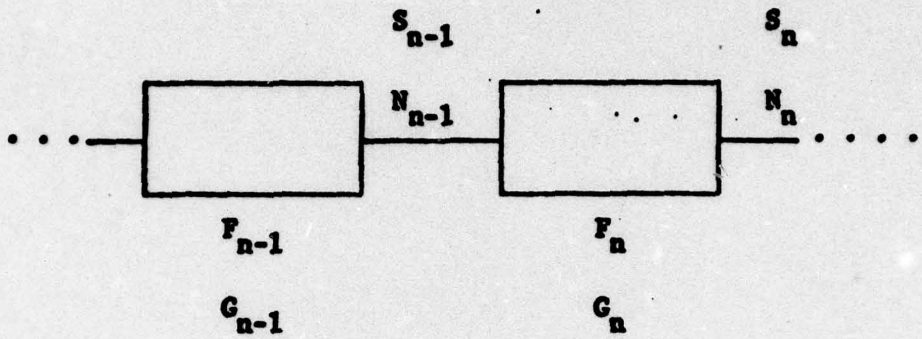


FIG. 2--Notation for chain of networks.

We then have Eqs. (3) to (5).

$$S_n = G_n S_{n-1} \tag{3}$$

$$N_{n-1} = T + \Delta N_{n-1} \tag{4}$$

$$N_n = F_n G_n T + G_n \Delta N_{n-1} \ . \tag{5}$$

Combining (3), (4), and (5) gives Eqs. (6) and (7).

$$\frac{S_n}{N_n} = \frac{S_{n-1}}{N_{n-1}} \rho_n \ . \tag{6}$$

Along the chain of networks, then, the signal to noise ratio at $n = b$ compared to that at $n = a$ is given by

$$\frac{S_b}{N_b} = \frac{S_a}{N_a} \prod_{n=a}^{b} \rho_n$$

$$\rho_n = \frac{1 + \frac{\Delta N_{n-1}}{T}}{F_{n-1} + \frac{\Delta N_{n-1}}{T}} \quad (7)$$

Let us now consider the case in which the noise input to the first network of the series $(n = 0)$ is Johnson noise $T$, i.e., $\Delta N_0/T = 0$. Then we have Eq. (8),

$$\begin{aligned}\frac{\Delta N_n}{T} &= (F_n G_n - 1) + G_n \frac{\Delta N_{n-1}}{T} \\ &= (F_n G_n - 1) + G_n (F_{n-1} G_{n-1} - 1) \\ &\quad + G_n G_{n-1} (F_{n-2} G_{n-2} - 1) \\ &\quad + \cdots + (G_n G_{n-1} \cdots G_2)(F_1 G_1 - 1) .\end{aligned} \quad (8)$$

If we assume the $F_n G_n \gg 1$ for all $n$, Eq. (8) becomes Eq. (9). If all networks are identical so that $F_n = F$ and $G_n = G$, then Eq. (9) becomes

$$\frac{\Delta N_n}{T} \sim F_n G_n + F_{n-1} G_n G_{n-1} + F_{n-2} G_n G_{n-1} G_{n-2}$$

$$+ \ldots + F_1 (G_n G_{n-1} \ldots G_1) \tag{9}$$

$$\frac{\Delta N_n}{T} \sim FG \left(1 + G + G^2 + \ldots + G^{n-1}\right)$$

$$= FG \frac{1 - G^n}{1 - G} \tag{10}$$

$$\rho_n = \frac{(1 - G) + FG\left(1 - G^{n-1}\right)}{F(1 - G) + FG\left(1 - G^{n-1}\right)}$$

$$= \frac{(1 - G) + FG\left(1 - G^{n-1}\right)}{F\left(1 - G^n\right)} \tag{11}$$

If we let $G \to \infty$ then, in Eq. (11), $\rho_n \sim 1$. In this case the noise figure of the system does not change as we go along the chain. In some

applications, the output of the chain in Fig. (1) would, in effect, be connected directly back to the input. We then inquire how many circulations an injected signal can make around this closed loop, for a given degradation in the signal-to-noise ratio, with the gain adjusted for a stable loop (no self-oscillation). In this case $G$ is limited in value to the neighborhood of unity and less. Thus put $G = 1 + \epsilon$, which gives for $\rho_n$, as $\epsilon \to 0$,

$$\rho_n \sim \frac{\epsilon + F(n-1)\epsilon}{F n \epsilon} = \frac{(n-1)F + 1}{nF} . \tag{12}$$

Let us evaluate the $S/N$ ratio after $n$ sections. We have from Eq. (12)

$$\pi\rho_n = \frac{F+1}{F} \cdot \frac{2F+1}{2F} \cdot \frac{3F+1}{3F} \cdots \frac{(n-1)F+1}{(n-1)F} \cdot \frac{1}{nF}$$

$$= f(F) \cdot \frac{1}{nF} \tag{13}$$

where $f(F) > 1$ and approaches 1 as $F$ becomes large. Thus putting $f(F) = 1$ underestimates the $S/N$ ratio, so that

$$\frac{(S/N)_n}{(S/N)_1} > \frac{1}{nF} \quad . \tag{14}$$

For example, if $F = 10$ (typical 10 dB noise figure), and if $n = 10^3$, then $\frac{1}{nF} = 10^{-4}$. In this case the S/N ratio is degraded by less than 40 dB in going through 1000 networks each having noise figure of 10 dB.